

Research article

# PREDICTIVE MODEL TO MONITOR TRANSITORY DEPOSITION IN SEMI CONFINED AQUIFERS INFLUENCED BY COMPRESSIBLE FLUID IN HETEROGENEOUS FORMATION IN OKIRIKA RIVERS STATE OF NIGERIA

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## Abstract

Predictive model to monitor the rate of transitory in semi confined aquifers influenced by compressible fluid in heterogeneous formation has been developed. The models were developed through formulated governing equation from influential variables in the system. The rate of transitory were found to deposit through geological setting in the study location, the rate of compressibility of fluid are influenced through formation characteristics in the study area , the formation in the study location were confirmed to have deposited semi confined aquifer, hydrogeological studies confirmed the rate of aquifer deposition thus, the level of transitory through the stratification of the formation in the study area. The developed mathematical model is imperative because it determines the rate of transitory deposition in semi confined aquifer; it has expressed the relation of formation characteristics with other variables in the system. This conditions influence the deposition of transitory in soil formation, experts will find the developed model useful in monitoring the level of fluid flow in soil under the influence of soil compressibility in heterogeneous formation. **Copyright © WJSTR, all rights reserved.**

**Keywords;** Predictive model, transitory, semi confined aquifers and heterogeneous formation

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## 1. Introduction

Modelling method is being used to study several aspects of soil behaviour. Flow velocity in a centrifuge model at will be times faster compared to the prototype it represents. Consequently the scaling law for seepage velocity has been established (Schofield, 1980) and has been confirmed experimentally (Arulanandan et al., 1988). This scaling law for seepage velocity has been accepted and commonly used, but the question of whether it is the Darcy's permeability (hydraulic conductivity) or the hydraulic gradient that is a function of gravity has not been addressed properly. This issue was highlighted by Goodings (1979), who points out to the multiplicity of the concepts in

scaling flow velocity. Butterfield (2000) and Dean (2001) also discussed this issue. Pokrovsky and Fyodorov (1968), Cargill and Ko (1983), Tan and Scott (1985) and more currently Singh and Gupta (2000) are surrounded by several others who have taken permeability ( $k$ ) to be straight proportional to gravity and hydraulic gradient ( $i$ ) to be autonomous of gravity. While this expressing why seepage velocity has a scaling law, there is an option clarification for the increase of seepage velocity in a centrifuge. Schofield (1980), Hussaini et al. (1981), Goodings (1984), and Taylor (1987) have all suggested that permeability to be autonomous of gravity and it is the hydraulic gradient which has got a scaling factor of  $N$ . Since both sides of the clarification result in the same final answer *and* it is the final seepage velocity that is taken it significant in many cases, the controversy has often been overlooked. In this technical note we attempt to resolve this controversy by using the energy gradient as the driving force on the pore fluid.

In gravel mole drains, apart from a transport function, the gravel has also a structural function. In wastewater treatment, gravel layers are employed in stratified sand filters (Nichols *et al.*, 1997). In groundwater hydrology, gravel layers are commonly exploited as aquifers and may also give rise to hillside seepage and springs (Selim and Kirkham, 1972). Gravel layers may also serve to transport contaminants with significant dispersion at high pore velocities (Pfankuch, 1963 quoted in Leij and van Genuchten, (1999).

In saturated soils,  $K$  values ranging from 10–4 m/s for sandy soils to 10–10 m/s for fine-grained compacted landfill clays are commonly found (Rawls, Brakensiek and Saxton, 1982; Olson and Daniel, 1981; Youngs, Leeds-Harrison and Elrick, 1995). Hydraulic gradients in saturated soils generally vary by only half an order of magnitude, although in the surface layers of unsaturated soils at low water contents hydraulic gradients may be very large, e.g. 10 to 1000 (Hosty and Mulqueen, 1996;. Thusyanthan &.Madabhushi 2003)

## 2. THEORETICAL BACKGROUND

Monitoring of flow path of fluid are based on several formation characteristics. To predict the transitory movement of fluid is through the flow paths. The interaction between surface water and groundwater bodies are determined by several hydrogeological setting under the influence of geological depositions in soil and water environment. To monitor the transitory flow of semi-confined beds influenced by compressibility of fluids is from geological setting, it confirms to deposit heterogeneous formation in the study location. The interaction of surface and groundwater observed different rate of fluid flows under the influence of structural deposition of heterogeneous soils. Semi-confined beds are influenced by compressibility of fluids through the formation of overburden pressure in soil. Such conditions are found to experience transitory flow from semi-impermeable layers or penetrating aquifers. This conceptual framework experienced in the study location where the hydrogeological setting observed to deposit transitory formations, structural depositions of the formation are confirmed in the study location, but not predominant in the study area. Semi-confined bed may be found to be influenced from these formations, because it is influenced by compressibility of fluids, the study of this direction of flow from the hydrogeological setting makes it imperative to develop a mathematical model that will monitor the transitory depositions in semi-unconfined beds,

hydrogeological studies has confirmed the influenced by compressibility of fluids deposited thus heterogeneous formation.

### 3. Governing Equation

$$\frac{\partial hi}{\partial x} \left( Kij \frac{\partial h}{\partial x} \right) = \frac{\partial h}{\partial t} + W \quad \dots\dots\dots (1)$$

Applying physical splitting techniques on equation (1) we have

The governing equation, through the application of physical splitting techniques applied to splitting of the variables, this is to determine there behaviour at several phase of the system, these applications are imperative because the functionalities of the variables will be expressed at various phase considered in the flowing process at various strata.

$$\frac{\partial h}{\partial x} = Kij \frac{\partial hi}{\partial x} \quad \dots\dots\dots (2)$$

$$\left. \begin{array}{l} x = 0 \\ h_{(o)} = ho \\ \frac{\partial hi}{\partial x} \Big|_{x=0} = 0 \end{array} \right\} \dots\dots\dots (3)$$

$$\frac{\partial h_2}{\partial x} = \frac{\partial h}{\partial t} + W \quad \dots\dots\dots (4)$$

$$\left. \begin{array}{l} x = 0 \\ t = 0 \\ h_{(o)} = ho \\ \frac{\partial h_3}{\partial x} \Big|_{x=0, \beta} = 0 \end{array} \right\} \dots\dots\dots (5)$$

$$Kij \frac{\partial h_3}{\partial h} = - \frac{\partial h_3}{\partial t} + W \quad \dots\dots\dots (6)$$

$$\left. \begin{array}{l} x = 0 \\ t = 0 \\ h_{(o)} = 0 \end{array} \right\} \dots\dots\dots (7)$$

Applying direct integration on (2)

$$\frac{\partial hi}{\partial x} = Kij h + Zi \quad \dots\dots\dots (8)$$

Again, integrating equation (8) directly, yields

$$h = Khx + Zix + Z_2 \dots\dots\dots (9)$$

Subject to equation (3), we have

$$ho = K_2 \dots\dots\dots (10)$$

And subjecting equation (8) to (3)

$$\text{At } \left. \frac{\partial hi}{\partial x} \right|_{x=0, h(o) = h_o} = 0$$

Yield

$$0 = Kh + Z_2$$

$$\Rightarrow K_1 = Kh_o \dots\dots\dots (11)$$

So that, we put (10) and (11) into (9), we have

$$hi = Kijhix - Kh(o)x + ho \dots\dots\dots (12)$$

$$hi - Kijhix = ho - Khox \dots\dots\dots (13)$$

$$\Rightarrow hi (-K, a) = ho (-Kx)$$

$$\Rightarrow hi = ho \dots\dots\dots (14)$$

Hence equation (14), entails that at any given distance, x, we have constant transitory flow in the systems

The formation are assumed to have developed a serious influence, this include permeability of the formation and the degree of void ratio, this parameters determine the rate of constant transitory flow in the strata, this were expressed in the system, the degree of void ratio deposited in the formation where found to play more role, due the deposition influence from the stated parameter.

Now, we consider equation (4), which is a linear exponential flow of the system

$$\frac{\partial h_2}{\partial x} = \frac{\partial h}{\partial t} + W \dots\dots\dots (4)$$

We approach this system by using the Bernoulli's method of separation of variables

$$\text{i.e. } h_2 = XT \dots\dots\dots (15)$$

$$\text{i.e. } \frac{\partial h_2}{\partial x} = X^1 T^1 \dots\dots\dots (16)$$

$$\frac{\partial h_2}{\partial t} + W = XT^1 \dots\dots\dots (17)$$

Put (16) and (17) into (15), so that we have

$$XT^1 = XT^1 W \dots\dots\dots (18)$$

$$\text{i.e. } \frac{X^1}{X} = \frac{T^1W}{T} = -\lambda^2 \quad \dots\dots\dots (19)$$

$$\text{Hence } \frac{X^1}{X} + \lambda^2 = 0 \quad \dots\dots\dots (20)$$

$$\text{That is, } X^1 + \lambda^2 X = 0 \quad \dots\dots\dots (21)$$

$$T^1W + \lambda^2 T = 0 \quad \dots\dots\dots (22)$$

$$\text{From (21), } X = \frac{A \cos \lambda}{\sqrt{X}} + \frac{B \sin \lambda}{\sqrt{\beta}} \quad \dots\dots\dots (23)$$

And (16) gives

$$\boxed{T = h \ell \frac{-\lambda^2}{W} t} \quad \dots\dots\dots (24)$$

The model expressed rate of transitory of flow under the application of plug flow system including time, but the rate of transitory varies depending the flow path, this condition are determine on the state of the formation, the condition of this strata implies that the sediments including its disintegration are determine from the geological setting, the flow path influenced by the structural deposition generating the flow to be mobile or immobile, this influence the rate of velocity with respect to flow, based on this condition time of transitory at different formation varies as it express on the model. Soil with different micropoles are with different hydraulic conductivity of fluid, this generate different time as express on the model phase of the studies

By substituting (23) and (24) into (15), we get

$$\boxed{h_2 = \left( A \cos \frac{\lambda}{\sqrt{x}} + B \sin \frac{\lambda}{\sqrt{x}} \right) h \ell \frac{-\lambda^2}{W} t} \quad \dots\dots\dots (25)$$

Expressing the subject relations through the substitution of (23) and (24) into (15) the model expression are correlated to interact with other variables under the influence of exponential condition, through the increase of velocity of flow, this condition can only be determined through the rate of flow path in the formation with respect to time and distance, these conditions determine the rate of transitory in the strata influenced by formation characteristics between the soil strata and groundwater aquifers.

Subject equation (25) to conditions in (5), so that we have

$$h_o = AC \quad \dots\dots\dots (26)$$

From equation (26) becomes

$$h_2 = h_o \ell \frac{-\lambda^2}{W} t \cos \frac{\lambda}{\sqrt{x}} \dots\dots\dots (27)$$

Again, at

$$\left. \frac{\partial h_2}{\partial x} \right|_{x=0, B} = 0, t = 0$$

Equation (27) becomes

$$\frac{\partial h_2}{\partial x} = \frac{\lambda^2}{\sqrt{x}} h_o \ell \frac{-\lambda^2}{W} t \sin \frac{\lambda}{x} \dots\dots\dots (28)$$

$$0 = -h_o \frac{\lambda}{\sqrt{x}} \sin \frac{\lambda}{\sqrt{x}} \dots\dots\dots (29)$$

i.e.

$$h_o \frac{\lambda^2}{\sqrt{x}} \neq 0 \text{ Considering flow path ways}$$

$$0 = -h_o \frac{\lambda^2}{\sqrt{x}} \sin \frac{\lambda}{\sqrt{x}} \beta \dots\dots\dots (30)$$

$$\Rightarrow \frac{\lambda}{\sqrt{x}} = \frac{n\pi}{2}, n = 1, 2, 3 \dots\dots\dots (31)$$

So that equation (27) becomes

$$h_2 = h_o \ell \frac{n^2 \pi^2 x}{2W} t \cos \frac{n\pi\sqrt{x}}{2} \dots\dots\dots (32)$$

$$\therefore \Rightarrow h_2 = -h_o \ell \frac{-n^2 \pi^2 x}{2W} t \cos \frac{n\pi}{2} x \dots\dots\dots (33)$$

The developed model in these phase express when there is high degree of permeability and porosity, there is the tendency of increase in transitory within the deposited formations, this condition generate high velocity and will definitely deposit high flow in semi confined bed, subject this state of the formation, the geological setting within the transitory zone experience high compressibility of fluid flow, it generate overburden pressure very fast, since the study location has a sand stone base in some region of the formation. This expressed model handle this condition on the rate of flow phase of the study, but formation characteristics such porosity at different degrees determine the rate of deposition of the transitory and the compressibility of fluid, thus in such heterogeneous formation asit expressed in the system, other variables in the flow phase also expressed there influence in the model stated in equation 33.

Now, we consider equation (6) which is the steady-flow state of the system

$$K_{ij} \frac{\partial h_3}{\partial x} = \frac{\partial h_3}{\partial x} + W \quad \dots\dots\dots (6)$$

Applying Bernoulli's method, we have

$$h_3 = XT \quad \dots\dots\dots (34)$$

$$\frac{\partial h_3}{\partial x} = X^1 T \quad \dots\dots\dots (35)$$

$$\frac{\partial h_3}{\partial t} = XT^1 \quad \dots\dots\dots (36)$$

Put (35) and (36) into (6), so that we have

$$K_{ij} X^1 T = XT^1 W \quad \dots\dots\dots (37)$$

$$\text{i.e. } K_{ij} \frac{X^1}{X} = - \frac{XT^1 W}{T} = \varphi \quad \dots\dots\dots (38)$$

$$K_{ij} \frac{X^1}{X} = \varphi \quad \dots\dots\dots (39)$$

$$- \frac{XT^1 W}{T} = \varphi \quad \dots\dots\dots (40)$$

$$\boxed{X = A \ell^{\frac{\varphi}{K_{ij} x}}} \quad \dots\dots\dots (41)$$

And

$$\boxed{T = \beta \ell^{\frac{-\varphi}{W t}}} \quad \dots\dots\dots (42)$$

the stated model in 42 and 43 shows the rate of fluid under the influences of the strata that experience transitory in semi confined bed, the formation also develop variation in hydrostatic pressure, the formation through high degree of porosity in some region experience overburden pressure in the strata, therefore 42 and 43 developed model in these phase monitor the rate of flow with respect to time and distance in semi unconfined bed.

Put (41) and (42) into (34), gives

$$h_3 = A \ell^{\frac{\varphi}{W x}} \beta \ell^{\frac{-\varphi}{W t}} \quad \dots\dots\dots (43)$$

$$h_3 = AB \ell^{(x-t) \frac{\varphi}{W}} \quad \dots\dots\dots (44)$$

Subject equation (44) to (7), yield

$$h_3 = (o) = h_o \dots\dots\dots (45)$$

So that equation (45), becomes

$$h_3 = h_o \ell^{(x-t)\frac{\rho}{W}} \dots\dots\dots (46)$$

Now assuming that at the steady state flow, there is no any sources of solute, our concentration here is zero, so that equation (46) become

$$h_3 = 0 \dots\dots\dots (47)$$

Therefore, solution of the system is of the form

$$h = h_1 + h_2 + h_3 \dots\dots\dots (48)$$

We now substitute (14), (33) and (47) into (48), so that we have the model of the form

$$h = h_o + h_o \ell^{\frac{-n^2\pi^2}{2W}t} \text{Cos} \frac{n\pi}{2}x \dots\dots\dots (49)$$

$$\boxed{h = h_o \left( 1 + \ell^{\frac{-n^2\pi^2}{2W}t} \text{Cos} \frac{n\pi}{2}x \right)} \dots\dots\dots (50)$$

The final expressed model in the study were able to monitor the rate of transitory in semi confined aquifer, this were developed at several phase, because the rate of flow are influenced by different conditions under formation characteristics in soil and water environment. These condition were expressed in the system to determine various influenced that leads to transitory flow condition in semi confined aquifers, the developed models in these phase were finally coupled together to generate final mathematical model, these express all the conditions through the parameters that is significant in the study, the derived model stated at 50 will definitely solve the problem of transitory condition in unconfined aquifers influenced by compressibility of fluid in the study area.

#### 4. CONCLUSION

Transitory in semi confined are determine by the deposition of soil stratification through the geological setting in the study area. Soil structural deposition determines the rate of transitory in semi confined bed through compressible fluid, structural deposition that deposit heterogeneous formation generate transitory, in some instances this condition is as a result of the geological setting that influence semi confine bed in the study location. To determine the rate of transitory in semi confined aquifers under the influence of compressible fluid in heterogeneous formation, mathematical models were developed through express variable from formation characteristics, the developed models were derived through formulated governing equations, the model were derived in phases according to the behaviour of fluid movement in soil, this is considered to develop transitory condition in semi confined aquifers in



the study area, the study is imperative because the rate of transitory in semi confined aquifers are through the influence of compressible fluid. Experts in water resources and environmental engineers including hydro geologist will find these model useful to monitor the rate of influence in transitory deposition in the study area. Compressible fluid through the deposition of over burden pressure were experienced in the study location can be determined through hydrogeological studies, this will be useful as professionals will apply the model to design ground water system that will generate optimum yield rate from such semi confined formation in the study location.

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